Enrollment No: ____________________ **Exam Seat No:____________________**

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name :Differential and Integral Calculus

Subject Code : 4SC04MTC1 Branch : B.Sc.(Mathematics/ Physics)

Semester : 4 Date :19/11/2015 Time :2:30 To5:30Marks : 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $div\ \vec{F} = 0$ then $\vec{F}$ is called irrotational vector function. True / False. **(01)**

b) Write formula of the radius of curvature for the polar curve $r = f(\theta)$. **(01)**

c) The transformation $x = r\cos\theta, y = r\sin\theta$transform the area element $dy\ dx$ in to $|J|\,dr\ d\theta$ where $|J|$ is equal to **(01)**

a) r b) r^2 c) $r^2\sin\emptyset$ d) None of these

d) Area of region R is **(01)**

a) $\int_R x\ dy$ b) $\int_R y\ dx$ c) $\iint_R dx\,dy$ d) $\iiint_R dx\,dy\,dz$

e) State Green's theorem. **(02)**

f) If $\vec{F} = 3t^2 i + 4t\,j + 4t^3 k,$ find value of $\int_{t=1}^{t=2} \overrightarrow{F\ (t)}\ dt$. **(02)**

g) Evaluate: $\int_0^2 \int_0^2 (x^2 + y^2)\ dx\ dy.$ **(02)**

h) Verify $u = \log(x^2 + y^2)$ is a solution of partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$ **(02)**

i) Evaluate: $\int_0^1 \int_0^x e^x\ dx\ dy.$ **(02)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Find the direction derivative of $\emptyset(x, y, z) = xy^2 + yz^3$ at the point (2 , -1, 1) in the direction of the vector $i + 2j + 2k.$ **(05)**

	b)	In usual notation prove that $div(\bar{f} \times \bar{g}) = \bar{g}\ curl\,\bar{f} - \bar{f}\ curl\,\bar{g}$.	**(05)**
	c)	If $\bar{f} = (ax+3y+4z)i + (x-2y+3z)j + (3x+2y-z)k$ is solenoidal then find a.	**(04)**
Q-3		**Attempt all questions**	**(14)**
	a)	Change the order of integration in the integral and then evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$.	**(05)**
	b)	Evaluate $\iint_R xy\,dy\,dx$ where R is the positive quadrant of the circle $x^2+y^2=a^2$.	**(05)**
	c)	Evaluate: $\int_0^1 \int_0^x \int_0^{\sqrt{x+y}} z\ dx\ dy\ dz$.	**(04)**
Q-4		**Attempt all questions**	**(14)**
	a)	Applying the transformations $u = x+y, y = uv$ and evaluate $\int_0^1 \int_0^{1-x} e^{\frac{y}{(x+y)}}\,dy\,dx$.	**(05)**
	b)	Using multiple integral, prove that volume of sphere having radius a is $\frac{4}{3}\pi a^3$.	**(05)**
	c)	Find $curl\,\overrightarrow{F}$, where $\vec{F} = grad(x^3+y^3+z^3-3xyz)$.	**(04)**
Q-5		**Attempt all questions**	**(14)**
	a)	Evaluate $\int_c \vec{F}\overrightarrow{dr}$ where $\vec{F} = x^2 i + xyj$ and c is the boundary of the square in the plane $z=0$ and bounded by the lines $x=0, y=0, x=a$ and $y=a$.	**(05)**
	b)	Find $\int_c \frac{dx}{x+y}$, where $C: x = at^2, y = 2at,\ 0 \le t \le 2$.	**(05)**
	c)	Evaluate $\int_{(1,1)}^{(2,3)} x\,dy$ over the line segment joining the points $(2,3)$ to $(1,1)$.	**(04)**
Q-6		**Attempt all questions**	**(14)**
	a)	If $\varphi = 2xyz^2,\ \vec{F} = xyi - zj + x^2k$ and c is the curve $x=t^2, y=2t, z=t^3$ from $t=0$ to $t=1$, evaluate the line integrals (a) $\int_c \varphi\,\overrightarrow{dr}$ (b) $\int_c \vec{F} \times \overrightarrow{dr}$.	**(06)**
	b)	Solve: $p\tan x + q\tan y = \tan z$.	**(04)**
	c)	Find multiple points of the curve $x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0$.	**(04)**
Q-7		**Attempt all questions**	**(14)**

a) Verify Green's theorem for the function $\overrightarrow{F} = (x+y)i + 2xy\,j$ and c is the rectangle in the $XY -$ plane bounded by $x = 0, y = 0, x = a, y = b$. **(07)**

b) Form the partial differential equation by eliminating the arbitrary constants or function. **(07)**

(i) $z = (x+a)(y+b)$

(ii) $f(x+y+z, x^2+y^2+z^2) = 0$

Q-8 **Attempt all questions** **(14)**

a) Prove that radius of curvature for the curve $y = f(x)$ is $\frac{(1+{y_1}^2)^{\frac{3}{2}}}{y_2}$. **(07)**

b) Verify Stock's theorem for $\vec{F} = x^2\,i - xy\,j$ where c is a rectangle in plane $z = 0$ and bounded by the lines $x = 0, y = 0, x = a, y = a$. **(07)**